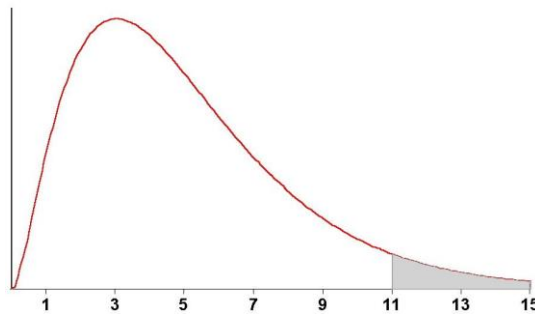


χ^2 (Chi-Square)

- one of the most versatile statistics there is
- can be used in completely different situations than “t” and “z”
- χ^2 is a **skewed** distribution



- Unlike z and t, the tails are not symmetrical.
- χ^2 can be used for many different kinds of tests.

We will learn 2 separate kinds of χ^2 tests.

Matrix Chi-Square Test

(a.k.a. “Independence” Test)

- Compares two qualitative variables, which are usually organized in a **table** (matrix)
- **QUESTION:** Does the distribution of one variable change from one value to the other variable to another.

EXAMPLES

- Are the colors of M&Ms different in big bags than in small bags?
- In an election, did different ethnic groups vote differently?
- Do different age groups of people access a website in different ways (desktop, laptop, smartphone, etc.)?

The information is generally arranged in a **contingency table** (matrix).

- If you can arrange your data in a table, a matrix chi-square test will probably work.

For example:

Suppose in a TV class there were students at all 5 ILCC centers, in the following distribution:

Center	Male	Female
Algona	5	7
E'burg	3	2
E'ville	4	4
Spenc.	4	7
S.L.	3	3

Does the distribution of men and women vary significantly by center?

- Our question essentially is—Is the distribution of the columns different from row to row in the table?
- A significant result will mean things **ARE** different from row to row.
- In this case it would mean the male/female distribution varies a lot from center to center.

The test process is still the same:

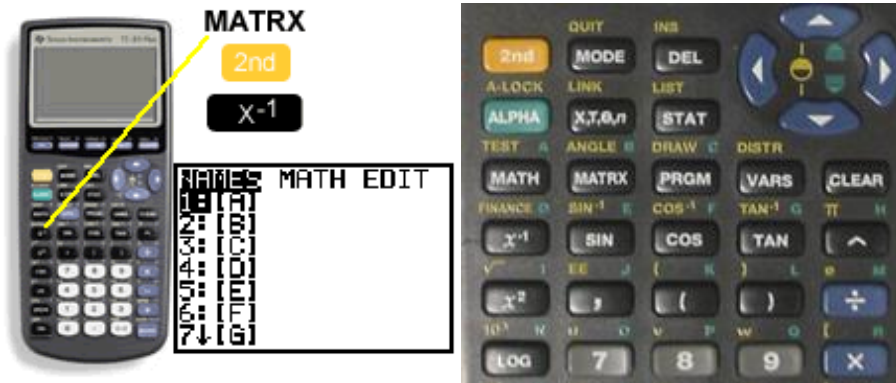
1. Compute a p-value.
2. Compare, and make a decision.

In this problem ...

- Since there's no α given in the problem, let's use $\alpha = .05$

1. Enter the observed matrix as [A] in the MATRIX menu.

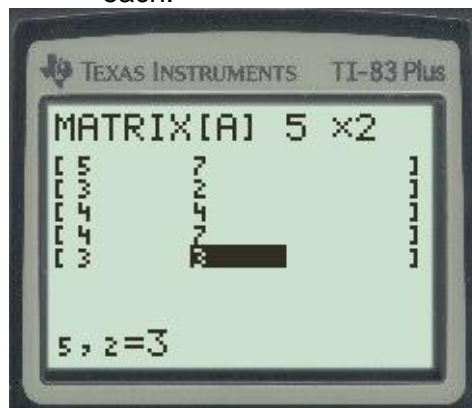
- Press **MATRX** or **2nd** and **X⁻¹**, depending on which TI-83/84 you have.



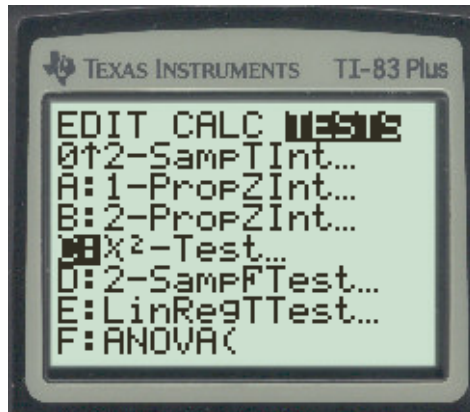
- Choose "EDIT" (use arrow keys)
- Choose matrix [A] (just press ENTER)



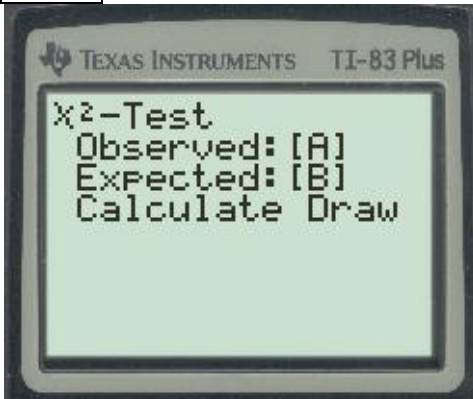
- Type the number of rows and columns, pressing **ENTER** after each.
- Enter each number, going across each row, and hitting **ENTER** after each.



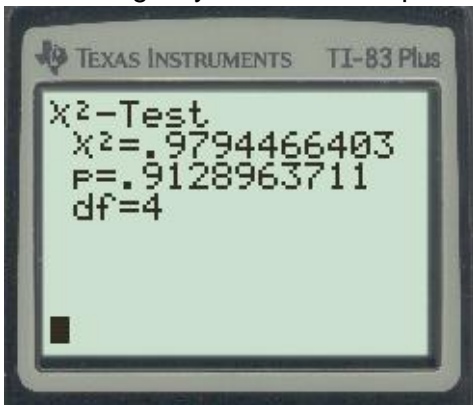
2. Press **2nd** and **MODE** to QUIT back to a blank screen.
3. Go to **STAT**, then **TESTS**, and choose **X²-Test** (easiest with up arrow)
(Note on a TI-84 this is "X²-Test", **not** "X²-GOF Test")



4. Make sure it says [A] and [B] as the observed and expected matrices. If it does just hit **ENTER** three times.



5. The read-out will give you χ^2 and the p-value (which is what you care about).



RESULT

- $.979 < 9.49$
- NOT significant

Categorical Chi-Square Test
(a.k.a. “Goodness of Fit” Test)



QUESTION:

- Is the distribution of data into various categories different from what is expected?
- Key idea—you have **qualitative** data (characteristics) that can be divided into **more than 2** categories.

EXAMPLES

- Are the colors of M&Ms distributed as the company says?
- Is the racial distribution of a community different than it used to be?
- When you roll dice, are the numbers evenly distributed?

You’re comparing what the distribution in different categories **should** be with what it actually is in your sample.

HYPOTHESES:

- H_1 : The distribution is significantly different from what is expected.
- H_0 : The distribution is not significantly different from what is expected.

SAMPLE PROBLEM:

You want to know if a die is fair.
You roll it 60 times and get 7 1’s, 6 2’s, 11 3’s, 15 4’s, 13 5’s, and 8 6’s.
At the .10 level of significance can you say the die isn’t fair?

This test is not built into the TI-83 (though you can download programs to do it). If you have a TI-84, here’s what you do ...

Enter the numbers

- Go to **STAT** → EDIT
- Type the **observed** values in L1.
- Type the **expected** values in L2.

Enter Observed (L1)

Enter Expected (L2)

L1	L2	L3	2
7	10	-----	
6	10		
11	10		
15	10		
13	10		
8	10		
-----	-----		
L2(1)=10			

Note that for L2 (expected) you can save time by ...

- If even distribution is expected, take the total divided by the number of categories.
- Otherwise, take each percent times the total.

Hit 2nd / **MODE** to QUIT back to a blank screen.

Do the test

- Go to **STAT** → TESTS
- Choose choice "D" (you may want to use the up arrow)... χ^2 GOF-Test

```
STAT → TESTS
D:  $\chi^2$  GOF-Test...
EDIT CALC TESTS
B: 2-PropZInt...
C:  $\chi^2$ -Test...
D:  $\chi^2$ GOF-Test...
E: 2-SampFTest...
F: LinRegTTest...
G: LinRegTInt...
H: ANOVA(
```

- Make sure Observed says L1 and Expected says L2.
- On the "df" line, enter 1 less than the number of categories.

Enter the lists, df
and Calculate

```
 $\chi^2$ GOF-Test
Observed: L1
Expected: L2
df: 5
Calculate Draw
```

- As with t and z tests, in the read-out, what you mostly care about is the p-value.

Output

```
 $\chi^2$ GOF-Test
 $\chi^2=6.4$ 
P=.2692187981
df=5
CNTRB=(.9 1.6 ...
```

RESULT

.292 > .10 → NOT SIGNIFICANT

EXAMPLE

You think your friend is cheating at cards, so you keep track of which suit all the cards that are played in a hand are. It turns out to be:

- ♦ → 4
- ♥ → 2
- ♣ → 13
- ♠ → 1

You'd normally expect that 25% of all cards would be of each suit. At the .01 level of significance, is this distribution significantly different than should be expected?

Test

STAT → EDIT

L1	L2	L3
4	5	-----
2	5	
13	5	
1	5	
-----	-----	

L2(5) =

2nd → MODE (QUIT)

STAT → TESTS → X²GOF-Test

```
X2GOF-Test
Observed:L1
Expected:L2
df:3
Calculate Draw
```

```
X2GOF-Test
X2=18
P=.001234098
CNTRB={ .2 1.8 ...
```

- P = .001234

RESULT:

- 18 > 11.34
- Significant

If you don't have a TI-84 that will do this test ...

One option is to enter a program that will do it for you.

```
Chi-Square Goodness of Fit Program

Select PRGM > NEW >
Enter > CHIGOF
:((L1-L2)2/L2)→L3
:sum(L3)→C
:dim(L2)-1→D
:Disp "CHI SQUARE ="
:Disp C
:Disp "DF ="
:Disp D
:χ2cdf(C,99999,D)
→P
:Disp "P-VALUE ="
:Disp P
:Stop
```

These directions are available in the printed notes, and writing this program is explained in detail at this YouTube link ... <https://www.youtube.com/watch?v=UGEukx2EaEk> .

It is also possible to download TI-84 emulators for phones (free) or computers (usually for a fee).

Yet another choice is to go to any of several online X^2 calculators, such as <http://vassarstats.net/csfit.html>

Category	Observed Frequency	Expected Frequency	Expected Proportion	Percentage Deviation	Standardized Residuals
A				----	----
B				----	----
C				----	----
D				----	----
E				----	----
F				----	----
G				----	----
H				----	----

Sums:

Observed Frequencies:

Expected Frequencies:

Expected Proportions:

Reset
Calculate

[Note that for $df=1$, the calculated value of chi-square is corrected for continuity.] [For $df=1$, this is the uncorrected value of chi-square.]

chi-square =

df =

P =

[P is non-directional]

ONE MORE EXAMPLE:

A teacher wants different types of work to count toward the final grade as follows:

- Daily Work → 25%
- Tests → 50%
- Project → 15%
- Class Participation → 10%

When points for the term are figured, the actual number of points in each category is:

- Daily Work → 175
- Tests → 380
- Project → 100
- Class Participation → 75

TOTAL POINTS = 730

Was the point distribution significantly different than the teacher said it would be? (Use $\alpha = .05$)

This time it's easiest to take each percent times the total for the expected values.

L1	L2	L3
175	.25*730	-----
380	.5*730	
100	.15*730	
75	.1*730	
-----	-----	
L2(5) =		

L1	L2	L3
175	182.5	-----
380	365	
100	109.5	
75	73	
-----	-----	
L2(5) =		

Since we have 4 categories, there are 3 degrees of freedom.

χ^2 GOF-Test $\chi^2=1.803652968$ $P=.6141403319$ $df=3$ $CNTRB=\{.308219...$
--

RESULT

.614 > .05, so NOT significant.

The division is roughly the same as what it was supposed to be.