Expected Value

- predicting a future average
- If an event happened over and over again, the expected value is the average value that would occur.

EXAMPLE 1

CUSTOMER SATISFACTION GUARANTEED

Any Questions Call 1-800-645-9242

www.pch.com
PUBLISHERS CLEARING HOUSE

OFFICIAL RULES ENCLOSED

$1,000.00 LOCAL PRIZE AWARD GUARANTEE

Publishers Clearing House guarantees to award 210 $1,000.00 prizes in a nationwide Giveaway. A prize will be awarded in your local TV viewing area (or "Designated Market Area" as defined by Nielsen Media Research). This award could take place in your city/town since it's within this Designated Market Area. The $1,000.00 winner from your local TV viewing area will be announced on August 31, 2006. We must locate any winner and have him or her execute an Affidavit of Eligibility within 30 days or we will select an alternate winner.

"YOUR INITIALS" PRIZE AWARD GUARANTEE
$67,600.00 in Prizes Up for Grabs

Publishers Clearing House guarantees to announce a winner of at least $100.00 with the same initials as your first and last name on August 31, 2006. In Giveaway No. 1079, prizes of $100.00 will be awarded via random drawing from among eligible entrants with initials that match each of the 676 unique combinations of two letters in the alphabet. The initial combination for the winner of Giveaway No. 1055 at Giveaway end will be ineligible for a $100.00 award. Unclaimed prizes will be awarded at random from among all eligible entrants at Giveaway end.

SF1285MO

SWEEPSTAKES FACTS

PRIZE/VALUE	GIVEAWAY NO.	END DATE	EST. ODDS OF WINNING
$10 Million	1025	*2/28/07	1 in 330,000,000
$1 Million	1055	8/21/06	1 in 106,000,000
$30,450.00 (1 @ $25,000.00, 1 @ $2,450.00, 1 @ $2,000.00, 1 @ $1,000.00)	1069	*12/31/06	1 in 13,000,000
$1,000.00	1078	8/21/06	1 in 86,000**
$100.00	1079	8/21/06	1 in 27,000***
$1,000.00	1135	8/21/06	1 in 4,000,000
$10,000.00	1066	*12/31/06	1 in 45,000,000

* Winner may be determined earlier. See Official Rules for details.
** Odds are an average of the estimated odds of winning from all 210 areas involved in this promotion (210 prizes of $1,000.00). Actual odds of winning for each area will fluctuate depending on population and response to this and other selected Bulletins.
*** Odds are an average of the estimated odds of winning from all first and last name initial combinations involved in this promotion (676 prizes of $100.00). Actual odds of winning will fluctuate depending on response to this and other selected Bulletins.

You Have Not Yet Won. All Entries Have The Same Chance Of Winning. The winner has not been identified. We don't know who the winner is. If you enter our Sweepstakes, your entry will have the same chance to win as every other entry.

Enter For Free. You don't have to buy anything to enter. Just mail the entry form included in this mailing or follow the instructions in the Official Rules.

Enter As Often As You Like. You don't have to wait for us to mail you an entry form. You may submit additional entries simply by writing us at: Publishers Clearing House, 101 Winners Circle, Port Washington, NY 11050. Each time you write to us you will be entered once into each ongoing Sweepstakes. Each entry request must be mailed separately.

Buying Won't Help You Win. Your chances of winning without a purchase are the same as the chances of someone who buys something. It would not be lawful to give any advantage to buyers in a Sweepstakes.

In the Publisher's Clearing House sweepstakes, you have the following probabilities of winning different prizes:

$10,000,000	$\frac{1}{330,000,000}$
$1,000,000	$\frac{1}{106,000,000}$
$30,450	$\frac{1}{13,000,000}$
$1,000	$\frac{1}{86,000}$
$100	$\frac{1}{27,000}$

a. What is the expected value of the sweepstakes?
b. Is it worth the cost (mathematically) of a 44¢ stamp to enter?

EXAMPLE 2

On a winter day, the Acu-Weather Computer gives the following chances of various amounts of snow for the next day:

Amount	Chance
0 inches	10%
½ inch	5%
1 inch	5%
2 inches	5%
3 inches	10%
4 inches	20%
6 inches	25%
8 inches	20%

If you are the meteorologist for a TV station, how much snow should you say we can expect?

EXAMPLE 3

At the Garrigan Gala, 200 people buy tickets for a special raffle. The grand prize is a new car with a value of around $20,000. There is also one winner of $2,500 cash, one winner of $1,000 cash, and two winners of $500 cash.

a. What is the expected value?
b. Tickets for this raffle cost $150 each. From a mathematical standpoint, is this a good deal?

HOW TO FIND EXPECTED VALUE

$n_1 \bullet P(n_1) + n_2 \bullet P(n_2) + n_3 \bullet P(n_3)$
$+ n_4 \bullet P(n_4) + \ldots$

In advanced math notation, it's $\sum x \bullet P(x)$.

- Multiply each possible amount times the probability of getting that amount
- Add up all the answers.

TO DO EXAMPLE 1:

`10000000*1/330000000+1000000*1/106000000+30450*1/13000000+1000*1/86000+100*1/2700 0 =` **`.0574109109`**

a. The expected value is about 5.7¢ (less than 6¢ per entry).
(This means if you entered every day for the rest of your life, most of the time you'd lose, but you might win enough to average out to winnings of 5.7¢ each time you entered.)
b. No, it's not worth paying a 44¢ stamp to enter.
(Since you win less than 6¢ on average, that's not enough to balance out the cost of a stamp.)

TO DO EXAMPLE 2:

`0*.10+.5*.05+1*.05+2*.05+3*.10+4*.20+6*.25+8*.20 =` **`4.375`**

So we can expect 4.375 inches of snow. The meteorologist would probably say something like "a little over four inches".

TO DO EXAMPLE 3:

Assuming it was possible to win multiple prizes, this would be …

`1/200*20000+1/200*2500+1/200*1000+2/200*500 =` **`122.5`**

a. The expected winnings are $122.50
b. No, since you'd only win back an average of $122.50 and you pay $150. (However, this is just about the best return you'd ever get on any raffle—because the car is donated.)

To practice, you can do these problems in your book:

PAGE 437 = #7 – 16

Problem 7

DVDs	0	1	2	3	4
Probability	.42	.36	.14	.05	.03

```
0*.42+1*.36+2*.14
+3*.05+4*.03
```
.91 (or about 1)

Problem 8

$\frac{2}{10} * 3 = .6$

Realistically, you'd say 1 defective battery.

Problem 9

Accidents	0	1	2	3	4
Probability	.935	.03	.02	.01	.005

```
0*.935+1*.03+2*.02
+3*.01+4*.005
```
.12

Problem 10

Quantity	10	11	12	13	14	15
Prob.	.05	.15	.25	.30	.20	.05

```
10*.05+11*.15+12*.25
+13*.30+14*.20+15*.05
```
12.6

Problem 11
```
0*.43+1*.19+2*.12
+3*.09+4*.04+5*.03
+6*.03+7*.02+8*.05
```
1.73

Problem 12
```
20*.05+22*.1+24*.35
+26*.3+28*.15+30*.05
```
25.1 (million dollars)

Problem 13
```
4.9*3/50+5.0*8/50
+5.1*12/50+5.2*14/50
+5.3*8/50+5.4*5/50
```
5.162
```
Or … (4.9*3+5.0*8
+5.1*12+5.2*14
+5.3*8+5.4*5)/50
```
5.162

Problem 14
```
(4.5*2+4.6*4+4.7*8
+4.8*20+4.9*14+5.0*12
+5.1*4)/64
```
4.84375

Problem 15

$$\frac{1}{5000} \bullet 2000 + \frac{1}{5000} \bullet 500 + \frac{3}{5000} \bullet 100 + \frac{10}{5000} \bullet 25$$

$.61

Since the cost of a ticket is $1, there is actually a net loss of **$.39**.

(This assumes, like the Gala raffle example, that you can win more than one prize. If you couldn't, you'd just reduce the denominator of the fraction by the number of exhausted prizes—so the second probability would be $^{1}/_{4999}$, the third $^{3}/_{4998}$, and the last $^{10}/_{4995}$. The final answer, however, is essentially the same.)

Problem 16

Since there is a .96 chance he will live, there is a .04 chance he will die (and the company will pay off on the policy).

`.04*20000 =` **$800**

This means the company can expect to pay out an average of $800 for customers like him, so they must charge a minimum premium of $800 to earn back their expected expenses.