

Finite Math → Logarithms (ANSWERS)

Solve for "x". Round answers that don't come out even to two decimal places.

1. $5^x = 1,220,703,125$
 $\log(1220703125) / \log(5) = \underline{13}$

5. $4^{3x} = 268,435,456$
 $\log(268435456) / \log(4) / 3 = \underline{4.66666\dots}$

2. $2^x = 524,288$
 $\log(524288) / \log(2) = \underline{19}$

6. $9^{5x} = 3,486,784,401$
 $\log(3486784401) / \log(9) / 5 = \underline{2}$

3. $6^x = 100$
 $\log(100) / \log(6) = \underline{2.57019}$

7. $1.025^{3x} = 8000$
 $\log(8000) / \log(1.025) / 3 = \underline{121.32099}$

4. $14.7^x = 135,900.75$
 $\log(135900.75) / \log(14.7) = \underline{4.39745}$

8. $5^{2x} = 400,000$
 $\log(400000) / \log(5) / 2 = \underline{3.1016}$

Answer these questions.

9. If you deposit \$300 at 4%, compounded monthly, how long will it take until the account is worth \$500?

$$500 = 300 \left(1 + \frac{.04}{12}\right)^{12t} \rightarrow \log(500/300) / \log(1 + .04/12) / 12 = \underline{12.7919 \text{ years}}$$

10. If you deposit \$4500 at 6%, compounded daily, how long will it take until the account is worth \$20,000?

$$20000 = 4500 \left(1 + \frac{.06}{365}\right)^{365t} \rightarrow \log(20000/4500) / \log(1 + .06/365) / 365 = \underline{24.863 \text{ years}}$$

11. If you deposit \$987,654 at 3% interest, compounded semiannually, how long will it take until the account is worth \$1,000,000?

$$1000000 = 987654 \left(1 + \frac{.03}{2}\right)^{2t} \rightarrow \log(1000000/987654) / \log(1 + .03/2) / 2 = \underline{.4172 \text{ years}}$$

12. How long does it take money to double at 5% interest, compounded quarterly?

$$2 = 1 \left(1 + \frac{.05}{4}\right)^{4t} \rightarrow \log(2) / \log(1 + .05/4) / 4 = \underline{13.9494 \text{ years}}$$

13. How long does it take money to double at 12% interest, compounded daily?

$$2 = 1 \left(1 + \frac{.12}{365}\right)^{365t} \rightarrow \log(2) / \log(1 + .12/365) / 365 = \underline{5.777 \text{ years}}$$

14. How long does it take money to double at 9% interest, compounded monthly?

$$2 = 1 \left(1 + \frac{.09}{12}\right)^{12t} \rightarrow \log(2) / \log(1 + .09/12) / 12 = \underline{7.73 \text{ years}}$$

15. How long does it take money to **triple** at 10% interest, compounded daily?

$$3 = 1 \left(1 + \frac{.1}{365} \right)^{365t} \rightarrow \ln \log(3) / \ln \log(1 + .1/365) / 365 = \underline{10.9876 \text{ years}}$$

16. How long does it take money to **triple** at 4% interest, compounded annually?

$$3 = 1 \left(1 + \frac{.04}{1} \right)^{1t} \rightarrow \ln \log(3) / \ln \log(1.04) = \underline{28.011 \text{ years}}$$

Solve.

17. A bank advertises an effective rate of 7.52%. If interest is compounded daily, what is the nominal rate?

This is backwards from the previous effective rate problems we did. We're given the effective rate (r_{eff}) and we have to find the nominal rate (r). You won't have to do this on a test, but it's not all that hard to do:

$$.0752 = \left(1 + \frac{.r}{365} \right)^{365} - 1$$

To solve this, first add 1.

$$1.0752 = \left(1 + \frac{.r}{365} \right)^{365}$$

What we need to do is get rid of the 365th power. We can do this by taking both sides to the $\frac{1}{365}$ power (which is the same as the 365th root).

$$1.000198668 = 1 + \frac{.r}{365}$$

Now subtract 1

$$.000198668 = \frac{.r}{365}$$

Finally multiply by 365

$$.0725138929 = r \rightarrow \text{So it's about 7.25\%}$$

18. A bank advertises an effective rate of 2%. If interest is compounded quarterly, what is the nominal rate?

$$.02 = \left(1 + \frac{.r}{4} \right)^4 - 1$$

To solve this, first add 1.

$$1.02 = \left(1 + \frac{.r}{4} \right)^4$$

What we need to do is get rid of the 4th power. We can do this by taking both sides to the $\frac{1}{4}$ power (which is the same as the 4th root).

$$1.004962932 = 1 + \frac{.r}{4}$$

Now subtract 1

$$.004962932 = \frac{.r}{4}$$

Finally multiply by 365

$$.0198517263 = r \rightarrow \text{So it's about 1.985\%}$$