

# STATISTICS

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## Summary of Statistical Tests

### One-Sample z-test

#### QUESTION IT ANSWERS:

- Is the mean of your sample significantly different from the mean of a population?

#### WHAT YOU'RE DOING:

- Comparing your sample with a long-established reference group (like the national average)

#### WHAT YOU NEED TO KNOW:

- actual mean of your sample ( $\bar{x}$  --  $\mu_1$  on some graphing calculators)
- comparison mean of the population ( $\mu - \mu_0$  on the TI-83)
- standard deviation of the population ( $\sigma$ )

#### HOW TO READ THE TABLE:

- Use the infinity ( $\infty$ ) row at the bottom of the t-table.
- Look up  $\alpha$  at the top, and read the value for "z" at the bottom.

#### ADVANTAGES/DISADVANTAGES:

- It's a simple and very accurate test.
- You need to know  $\sigma$  (the standard deviation of the population, which you almost never actually know).

#### EXAMPLE:

- The average IQ score in this class is 114. The average IQ in America is 100, with a standard deviation of 15. Is this class significantly above average in intelligence?

#### GIVEWAYS TO LOOK FOR:

- Question asks about **average**
- You know **parameters** ("US Census" or "United Nations data") or you have a **big** sample

### One-Sample t-test

#### QUESTION IT ANSWERS:

- Is the mean of your sample significantly different from what you expect it should be?

#### WHAT YOU'RE DOING:

- Comparing an actual outcome with an expected outcome.

#### WHAT YOU NEED TO KNOW:

- actual mean of your sample ( $\bar{x}$  --  $\mu_1$  on some graphing calculators)
- what you expect the mean should be—or what it was in the past ( $\mu - \mu_0$  on the TI-83)
- standard deviation of the sample ( $s$ )

#### HOW TO READ THE TABLE:

- Find the degrees of freedom ( $n - 1$ ) and the level of significance ( $\alpha$ ) in the row and column headers of the table.
- The number where the  $df$  row and the  $\alpha$  column intersect is your "t" value.

#### ADVANTAGES/DISADVANTAGES:

- It's easy to find all the information you need to know ahead of time.
- The test is slightly less accurate and precise as a z-test.

#### EXAMPLE:

- An insurance company says the average claim for hail damage is \$487.43. After a severe storm, 23 customers file claims. The average amount of these claims is \$564.32, and the standard deviation is \$53.20. Were the claims for this storm significantly higher than normal?

#### GIVEWAYS TO LOOK FOR:

- Question asks about **average**
- You have a **small** sample ( $< 30$ )

## Two-Sample t-test

### QUESTION IT ANSWERS:

- Is the mean of one sample significantly different from the mean of another sample?

### WHAT YOU'RE DOING:

- Comparing two separate groups (such as men and women, or blacks and whites)

### WHAT YOU NEED TO KNOW:

- actual mean of each sample ( $\bar{x}_1$  and  $\bar{x}_2$ )
- standard deviation of each sample ( $s_1$  and  $s_2$ )
- the number in each sample ( $n_1$  and  $n_2$ )
- (If you expect that the means will be different—because, for instance, you gave a pre-test and they were different then—you can include the expected difference ( $\mu_1 - \mu_2$ ) as well. We did **NOT** discuss this in class at all.)

### HOW TO READ THE TABLE:

- Find the degrees of freedom ( $n_1 + n_2 - 2$ ) and the level of significance ( $\alpha$ ) in the row and column headers of the table.
- The number where the  $df$  row and the  $\alpha$  column intersect is your “t” value.

### ADVANTAGES/DISADVANTAGES:

- This is by far the easiest and most versatile way to compare two groups.
- You are assuming that your samples are random, which is not necessarily true.
- Large samples almost always give significant results on a t-test, sometimes even if the significant result is irrelevant or unimportant.

### EXAMPLE:

- On the SAT verbal test, a sample of 15 women had an average score of 456, with a standard deviation of 44. A sample of 12 men had an average score of 385, with a standard deviation of 55. Did the women do significantly better than the men?

### GIVEWAYS TO LOOK FOR:

- The problem has 2 averages and 2 standard deviations.

## One Proportion z-test

### QUESTION IT ANSWERS:

- Is the percentage of subjects in a sample with a given binomial characteristic significantly different from what you expect it should be?

### WHAT YOU'RE DOING:

- Comparing an actual percentage to an established benchmark.

### WHAT YOU NEED TO KNOW:

- actual number in the sample with the given characteristic ( $x$ )
- total number in the sample ( $n$ )
- (If you aren't using a TI-83, you will use these numbers to find  $p$ , which stands for the percent with the characteristic.)
- The number you expect the percentage should be ( $\hat{p}$ )

### HOW TO READ THE TABLE:

- Use the infinity ( $\infty$ ) row at the bottom of the t-table.
- Look up  $\alpha$  at the top, and read the value for “z” at the bottom.

### ADVANTAGES/DISADVANTAGES:

- This is a fairly easy way to see if a percentage is reasonable.
- You are assuming that your samples are random, which is not necessarily true.

### EXAMPLE:

- When it awards contracts, a government department is supposed to give preference to companies whose employees are at least 15% minority. One company has 73 employees, 13 of whom are minority. Is this significantly above the government target?

### GIVEWAYS TO LOOK FOR:

- **Percents** or the words “**out of**” are giveaways for **PROPORTION**.
- **ONE** proportion compares a group to what it’s supposed to be.

## Two Proportion z-test

### QUESTION IT ANSWERS:

- Is the percentage of subjects with a certain binomial characteristic significantly different in two different samples?

### WHAT YOU’RE DOING:

- Comparing the percentage of something in two different groups.

### WHAT YOU NEED TO KNOW:

- actual number in each sample with the given characteristic ( $x_1$  and  $x_2$  – OR –  $r_1$  and  $r_2$ )
- total number in each sample ( $n_1$  and  $n_2$ )
- (If you aren’t using a TI-83, you will use these numbers to find  $p_1$  and  $p_2$ , which stand for the percent in each sample with the characteristic.)
- (If you aren’t using a TI-83, you will also need to find  $p_p$  and  $q_p$ , the pooled proportions (in both samples together) with and without the characteristic.)
- (If you ARE using a TI-83, you will often need to use  $p_1$  and  $p_2$  and  $n_1$  and  $n_2$  to find  $x_1$  and  $x_2$ .)

### HOW TO READ THE TABLE:

- Use the infinity ( $\infty$ ) row at the bottom of the t-table.
- Look up  $\alpha'$  at the top, and read the value for “z” at the bottom.

### ADVANTAGES/DISADVANTAGES:

- This is a fairly easy way to compare two percentages.
- You are assuming that your samples are random, which is not necessarily true.

### EXAMPLE:

- In a political poll, 46% of men and 49% of women say they support Smith for President. If the poll included 247 men and 185 women, is the level of support for Smith significantly different for men and women?

### GIVEWAYS TO LOOK FOR:

- **Percents** or the words “**out of**” are giveaways for **PROPORTION**.
- **TWO** proportion compares the percent in 2 different groups.

## Standard Deviation $\chi^2$ Test

### QUESTION IT ANSWERS:

- Is the standard deviation of a sample significantly different than its expected value.

### WHAT YOU’RE DOING:

- Trying to see if things are too spread out.

### WHAT YOU NEED TO KNOW:

- the standard deviation of the sample ( $s$ )
- the expected (or past) standard deviation ( $\sigma$ )
- the number in the sample ( $n$ )

### HOW TO READ THE TABLE:

- Find the degrees of freedom ( $n - 1$ ) and the level of significance ( $\alpha$ ) in the row and column headers of the table.
- The number where the  $df$  row and the  $n$  column intersect is your “ $\chi^2$ ” value.
- (In the rare event that a problem asks if the standard deviation is LESS than normal, you will use  $1 - \alpha$ , rather than  $\alpha$  for your level of significance. We did not discuss this in any meaningful way in class.)

### **ADVANTAGES/DISADVANTAGES:**

- This is just about the easiest test there is.
- Except for a few quality control problems, you almost never want to know what this test tells you.

### **EXAMPLE:**

- A packing machine is supposed to create packages with an average weight of 453 grams. It is acceptable for the standard deviation to be 11 grams. A sample of 12 packages showed a standard deviation of 14 grams. Is the machine out of control?

### **GIVEWAYS TO LOOK FOR:**

- The **question** asks about **standard deviation** or **“too spread out”**
- You have **2 standard deviations** in the problem (but 1 or 0 averages).

## **Categorical (Multinomial or Goodness of Fit) $\chi^2$ Test**

### **QUESTION IT ANSWERS:**

- Is the distribution of a multinomial event significantly different than what is expected?

### **WHAT YOU'RE DOING:**

- You're testing one sample that can be divided into **more than two** categories.
- You're trying to see if the distribution among the categories is more than what it should be.

### **WHAT YOU NEED TO KNOW:**

- actual number in the sample that fall into each category (**O's** – observed values)
- number you'd expect to fall into each category (**E's** – expected values, which you often must find by multiplying a percentage times the total number in the sample)

### **HOW TO READ THE TABLE:**

- Find the degrees of freedom ( $k - 1$ , where “k” means the number of categories) and the level of significance ( $\alpha$ ).
- The number where the *df* row and the  $\alpha$  column intersect is your “ $\chi^2$ ” value.

### **ADVANTAGES/DISADVANTAGES:**

- This is the easiest way to compare things with more than two possibilities.
- If you don't know “E” values to compare to, you make the assumption that all categories should be divided evenly, which is not necessarily true.

### **EXAMPLE:**

- A teacher wanted to divide the points in her class so that 30% of the points came from daily work, 45% from tests, and 25% from class participation. At the end of the semester, there were 143 daily work points in the class, 195 test points, and 80 class participation points. Is the actual distribution of points significantly different than the expected value?

### **GIVEWAYS TO LOOK FOR:**

- The problem has **3 or more categories**

## **Matrix (Table or Independence) $\chi^2$ Test**

### **QUESTION IT ANSWERS:**

- Do the rows of a matrix have significantly different column distributions?

### **WHAT YOU'RE DOING:**

- You're analyzing a table of data, to see if a distribution among different categories is different for various groups.

### **WHAT YOU NEED TO KNOW:**

- actual number in the sample that fall into each cell of the table (**O's** – observed values)
- number you'd expect to fall into each category (**E's** – expected values, which you usually find for each cell of the table by taking the row total times the column total, divided by the grand total)
- (NOTE: Occasionally you will want to compare two tables, usually showing identical data at different times. Here the old table is your **E's**, and the new table is your **O's**. We did NOT do this variation in class.)

### HOW TO READ THE TABLE:

- Find the degrees of freedom  $(r - 1)(c - 1)$ , where “r” is the number of rows and “c” is the number of columns, and level of significance ( $\alpha$ ).
- The number where the  $df$  row and the  $\alpha$  column intersect is your “ $\chi^2$ ” value.

### ADVANTAGES/DISADVANTAGES:

- This is just about the only way to compare many variables at the same time.
- Depending on the size of your tables, this can be a very tedious test to complete.
- The results will tell you whether a result is significant or not, but they won’t tell you what there is in the distribution that makes it significant.

### EXAMPLE:

- B. Dalton’s keeps track of the type of purchases made by their male and female customers on a given day. The results are given below:

	Fiction Books	Non-Fiction	Audio Books	Magazines	Other
Men	102	83	17	14	5
Women	173	66	20	38	15

Is the distribution of purchases significantly different for men than for women?

### GIVEWAYS TO LOOK FOR:

- The problem has a **table** (or information that can be written in a table).

## Correlation r Test

### QUESTION IT ANSWERS:

- Is there a significant linear correlation between two variables?

### WHAT YOU’RE DOING:

- As one thing changes, does something else change in a predictable way?

### WHAT YOU NEED TO KNOW:

- actual correlation coefficient for the sample ( $r$ )
- number of ordered pairs in the sample ( $n$ )

### HOW TO READ THE TABLE:

- The number where the  $n$  row and the  $\alpha$  column intersect is your “r” value.

### ADVANTAGES/DISADVANTAGES:

- This is the only way there is to test whether a correlation is significant.
- If you don’t have a graphing calculator, you must either use a VERY complex formula to find “r” or you must make a rough estimate using the length and width of the rectangle that encloses the scatterplot.

### EXAMPLE:

- As cars get older, their trade in value gets less. A study of 29 cars found this to be true, with a correlation value of  $r=0.58$ . Is there a significant correlation between the age of a car and its trade-in value?

### GIVEWAYS TO LOOK FOR:

- Words like “**goes up**”, “**goes down**”, “**increases**”, and “**decreases**” generally indicate correlations.

## Sign Test

### QUESTION IT ANSWERS:

- Is there a significant difference between the median of a sample and its expected median?

### WHAT YOU’RE DOING:

- Seeing whether more things in a sample are above a certain number than below it.
- Seeing whether more things in a sample went up over time than went down.
- Seeing whether more things in a sample fit into one group than another.

### WHAT YOU NEED TO KNOW:

- how many things in your sample fit into the “+” and “-“ groups ( $n_1$  and  $n_2$ )
- total number in the sample ( $n$ )

### HOW TO READ THE TABLE:

- The number where the  $n$  row and the  $\alpha$  column intersect is your critical value.
- This test works BACKWARDS from all the others:
  - If the smaller of the two groups has a value **LESS** than the critical value, your result is significant.
  - Otherwise, the result is NOT significant.

### ADVANTAGES/DISADVANTAGES:

- This is probably the quickest, easiest test there is.
- You are making no assumptions whatsoever about the population or the kind of sample you have.
- While you will get a result that says “significant” or “not significant”, there is no meaningful number you can compare, nor can you say WHY the results are the way they are.

### EXAMPLE:

- The stock market went up on 12 days this month and down on 7 days. There was one day the market was unchanged. Did it go up significantly more than it went down?

### GIVEWAYS TO LOOK FOR:

- You’re just **counting** how many are in two groups.

## Runs (Randomness of Data) Test

### QUESTION IT ANSWERS:

- Are there significantly too many or too few runs in a set of data?

### WHAT YOU’RE DOING:

- Is the data random?

### WHAT YOU NEED TO KNOW:

- the number in each of two types (such as odd and even) into which the data might fall ( $n_1$  and  $n_2$ )
- how many runs (groups of one or more of the same type of data in a row) there are (there is no variable for this).

### HOW TO READ THE TABLE:

- Find “ $n_1$ ” and “ $n_2$ ” in the table as directed.
- Where the row and column intersect, there will be **two** numbers. These are the boundaries for the critical range.
- To do the test:
  - If the actual number of runs is within the critical range, the data is random (which technically means NOT a significant result).
  - If the actual number of runs is outside the critical range, the data is NOT RANDOM (which technically is a significant result).

### ADVANTAGES/DISADVANTAGES:

- This is the only easy test for randomness.
- You are making no assumptions whatsoever about the population or the kind of sample you have.
- While you will get a result that says “random” or “not random”, there is no meaningful number you can compare, nor can you say WHY the results are the way they are.

### EXAMPLE:

- A coin is flipped repeatedly and yields the following results:

H	T	H	H	H	T	T	T	T	T	T	H
---	---	---	---	---	---	---	---	---	---	---	---

Is this a fair coin? (Is there a random distribution of heads and tails?)

### GIVEWAYS TO LOOK FOR:

- The question asks about **“random”**.

# Which Test Should I Use?

- \_\_\_\_\_ 1. A music store believes that the distribution of music they sell on CD (rock, rap, alternative, country, classical, etc.) is different from the distribution they distribute via downloads. They make a table to record what types of music are sold in different formats.
- |                               |   |
|-------------------------------|---|
| <b>A. 2-sample t-Test</b>     | <b>C. Matrix <math>\chi^2</math> Test</b> |
| <b>B. 2-proportion z-Test</b> | <b>D. Runs Test</b>                       |
- \_\_\_\_\_ 2. According to the U.S. Census, the average age of all Americans is 34.2, with a standard deviation of 12.1 years. You think that the average age of people in Whittemore is older than that.
- |                           |                               |
|---------------------------|-------------------------------|
| <b>A. 1-sample z-Test</b> | <b>C. Correlation r-Test</b>  |
| <b>B. 1-sample t-Test</b> | <b>D. 1-proportion z-Test</b> |
- \_\_\_\_\_ 3. A salesman has a quota of selling 20 pairs of shoes per day. His supervisor believes the salesman is below his quota more often than he is above it.
- |                              |   |
|------------------------------|---|
| <b>A. 2-sample t-Test</b>    | <b>C. Matrix <math>\chi^2</math> Test</b> |
| <b>B. Correlation r-Test</b> | <b>D. Sign Test</b>                       |
- \_\_\_\_\_ 4. On average, it takes 2 hours for a drug to disperse throughout a patient's blood system. The standard deviation is supposed to be 5 minutes. In a recent hospital study, it was found that the standard deviation for drug dispersal was 9 minutes. The researcher thinks this is too large of a standard deviation.
- |   |                               |
|---|-------------------------------|
| <b>A. Runs Test</b>                                   | <b>C. 1-sample t-Test</b>     |
| <b>B. Standard Deviation <math>\chi^2</math> Test</b> | <b>D. 1-proportion z-Test</b> |
- \_\_\_\_\_ 5. Chevy claims that its cars get better gas mileage, on average, than Ford's cars do. *Consumer Reports* takes samples of the gas mileage from both lines of cars, and they figure the average and standard deviation for both Chevy and Ford.
- |                           |   |
|---------------------------|---|
| <b>A. 2-sample t-Test</b> | <b>C. 2-proportion z-Test</b>                         |
| <b>B. Sign Test</b>       | <b>D. Standard Deviation <math>\chi^2</math> Test</b> |
- \_\_\_\_\_ 6. A social scientist believes that drug use is highest in low-income neighborhoods. She wants to find out whether drug use goes up as family income goes down.
- |   |                              |
|---|------------------------------|
| <b>A. 1-sample z-Test</b>                             | <b>C. Correlation r-Test</b> |
| <b>B. Standard Deviation <math>\chi^2</math> Test</b> | <b>D. Sign Test</b>          |
- \_\_\_\_\_ 7. An insurance company has records for all its customers showing that 40% of all claims for doctor's office visits are "small", 45% are "moderate" and 15% are "large". They take a sample of recent claims at the local medical clinic and find that 60 were "small", 180 were "moderate", and 60 were "large". They believe the distribution of claims at the local clinic is too heavy on moderate and large claims, and they want to test to see if they can remove the clinic from their "preferred provider" list.
- |                              |  |
|------------------------------|--|
| <b>A. Correlation r-test</b> | <b>C. Categorical <math>\chi^2</math> Test</b> |
| <b>B. 1-sample t-Test</b>    | <b>D. 2-sample t-Test</b>                      |
- \_\_\_\_\_ 8. In 1978 a class-action suit was filed against the Illinois Lottery Commission. It alleged that what was then called the "Daily Game" (now "Pick Three") had been rigged. The lottery commission claimed that all the numbers were randomly selected. Tests were performed to see if the lottery numbers were indeed random.
- |  |                           |
|--|---------------------------|
| <b>A. Correlation r-Test</b>                   | <b>C. 2-sample t-Test</b> |
| <b>B. Categorical <math>\chi^2</math> Test</b> | <b>D. Runs Test</b>       |
- \_\_\_\_\_ 9. The NASDAQ exchange advertises that it is "the market of the future" and implies that the companies it lists do better than the more traditional New York Stock Exchange. To test this out, you take a sample of stocks from each exchange and record the mean and the standard deviation.
- |                               |   |
|-------------------------------|---|
| <b>A. 2-proportion z-Test</b> | <b>C. 2-sample t-Test</b>                 |
| <b>B. Sign Test</b>           | <b>D. Matrix <math>\chi^2</math> Test</b> |
- \_\_\_\_\_ 10. Zach was recently denied admission to the Florence Nightengale School of Nursing. He believes he is fully qualified and that the school discriminated against him because he was a man. He does some research and finds out that 23% of the applicants to Florence Nightengale are men. He wants to see if the percentage of males among the applicants who are accepted is significantly less than 23%.
- |                           |                               |
|---------------------------|-------------------------------|
| <b>A. 1-sample t-Test</b> | <b>C. 1-proportion z-Test</b> |
| <b>B. 2-sample t-Test</b> | <b>D. 2-proportion z-Test</b> |

- \_\_\_\_\_ 11. Shortly after the first O.J. Simpson trial, *Time* magazine surveyed both blacks and whites around the country. Only 40% of 1,200 whites surveyed felt the “not guilty” verdict was correct. On the other hand, out of 400 blacks surveyed, 73% felt it was the correct verdict. *Time* reported that there was a significant difference in the percentage of blacks and whites who felt the jury gave the correct verdict.
- A. 2-sample t-Test                      C. Sign Test  
B. 2-proportion z-Test                D. Runs Test
- \_\_\_\_\_ 12. Ninety-five customers at Harry’s Bar were given a blind taste test to see whether they liked Miller or Budweiser beer better. It turned out that 31 liked Miller better, 19 preferred Budweiser, and 55 either had no preference or couldn’t tell the difference.
- A. 2-sample t-Test                      C. Categorical  $\chi^2$  Test  
B. Correlation r-Test                 D. Sign Test
- \_\_\_\_\_ 13. In the early 1990s, the New York City Transit Commission made a major push to clean up the New York subway. They noticed that as the number of graffiti-covered cars went down, so did the number of crimes committed on the trains.
- A. Correlation r-Test                    C. 2-sample t-Test  
B. Matrix  $\chi^2$  Test                      D. 2-proportion z-Test
- \_\_\_\_\_ 14. Edna plays bingo every Wednesday at St. Jerome’s Parish Hall. After not winning a single game for six straight weeks, she starts to think Father Mulcahey isn’t picking the numbers randomly.
- A. Categorical  $\chi^2$  Test                 C. Standard Deviation  $\chi^2$  Test  
B. Matrix  $\chi^2$  Test                      D. Runs Test
- \_\_\_\_\_ 15. Julie works at the McDonalds in Spencer. She reads in a company brochure that 72% of customers order fries with their meal. She wants to know if the percentage who order fries in Spencer is higher than that.
- A. 2-proportion z-test                 C. Matrix  $\chi^2$  Test  
B. 1-sample z-Test                     D. 1-proportion z-Test
- \_\_\_\_\_ 16. Iowa Lakes wants to know if their student body truly represents the area they serve. They find out what percentage of the population of the region lives in Clay, Dickenson, Palo Alto, Emmet, and Kossuth Counties, and then they look at how many students come from each of those counties. They want to know if the distribution of students is different from what would normally be expected.
- A. Standard deviation  $\chi^2$  Test       C. Matrix  $\chi^2$  Test  
B. Categorical  $\chi^2$  Test                 D. Runs Test
- \_\_\_\_\_ 17. In order to fit properly, an auto part must be within a certain tolerance. The machine that manufactures the parts normally produces parts that are acceptably close to the ideal size. Occasionally, though, it will get out of adjustment and produce a range of sizes that are too spread out. The company wants to know if the sizes are too spread out and they have to adjust the machine.
- A. 1-Sample t-Test                      C. Standard Deviation  $\chi^2$  Test  
B. 2-Proportion z-Test                 D. Correlation r-Test
- \_\_\_\_\_ 18. A university is investigating a professor for sexist grading policies. One of the things they do as part of their investigation is to compare the grade distribution for men and women in the professor’s classes. They organize their data in the following table:
- |       |   |   |   |   |   |
|-------|---|---|---|---|---|
|       | A | B | C | D | F |
| Men   |   |   |   |   |   |
| Women |   |   |   |   |   |
- A. 2-Sample t-Test                      C. Matrix  $\chi^2$  Test  
B. 2-Proportion z-Test                 D. Runs Test
- \_\_\_\_\_ 19. An insurance company wants to see if their suggested hospital stay for a certain kind of surgery is in line with doctors’ current thinking. They ask a large sample of several hundred doctors their opinion on the appropriate length of stay and see if the average of the doctors’ is different than the current stay the insurance company suggests.
- A. 1-sample z-Test                      C. 1-proportion z-test  
B. 1-sample t-Test                      D. 2-proportion z-test
- \_\_\_\_\_ 20. One day a convenience store clerk counts how many customers pay with cash and how many use electronic payments to see if there are significantly more of one payment method than another.
- A. 1-Sample t-Test                      C. Sign Test  
B. 2-Sample t-Test                      D. Correlation r-Test