Example

LOS ANGELES TIMES – February 4, 2005

IQ as a Matter of Life, Death

- Anderson Hawthorne has been convicted of murdering a rival gang member in Los Angeles.
- He has been sentenced to death.
- His lawyer is appealing the sentence, saying that Hawthorne is mentally retarded.
- It is illegal to execute the mentally retarded in the United States.

QUESTION …
Is he retarded?

- The defendant's IQ has been measured at 71.
- The state used to define 80 or less as "mentally retarded.
- Due to the stigma attached to that label, they recently changed the definition to 70 or lower.
- The defendant's IQ is 71.
- For IQ, the average is 100, and the standard deviation is 15.

Consider

- What percent of all people have an IQ less than 80?
- … less than 70?
- … less than 71?

Problem ➔
You are a college admissions counselor, considering two students for admission. Jolene got a score of 25 on the ACT, while Roger got a score of 1600 on the new version of the SAT. Who did better on their college placement test?

To answer both of these questions, we will use **standard scores** (or z-scores)

- Tell how many standard deviations a score is away from the mean.

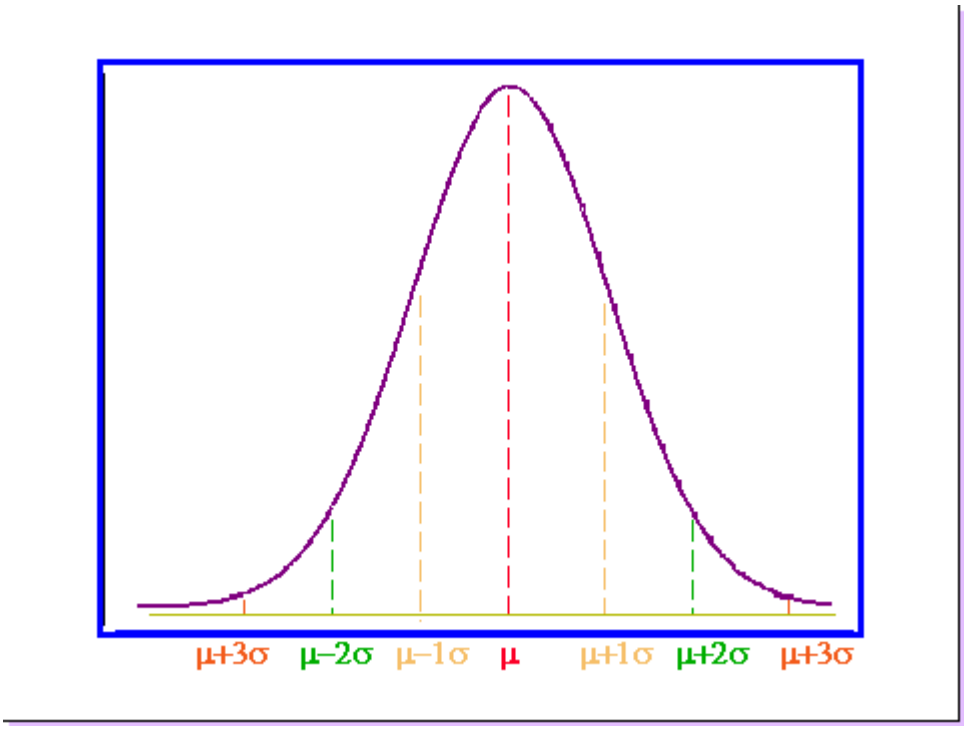
$$z = \frac{(x - \bar{x})}{s} \text{ or } z = \frac{(x - \mu)}{\sigma}$$

- A z-score of 0 is average
- **Positive** z-scores are **above** average
- **Negative** z-scores are **below** average

Empirical Rule

In any distribution that is approximately normal:

- 68% of the data is within 1 S.D. either side of the mean
- 95% of the data is within 2 S.D.s either side of the mean
- 99.7% of the data is within 3 S.D.s either side of the mean



68%

95%

99.7%

- So, … using the empirical rule:
  - 68% of all data are between z = -1 and z = 1
  - 95% of all data are between z = -2 and z = 2
  - 99.7% of all data are between z = -3 and z = 3

Problem ➔

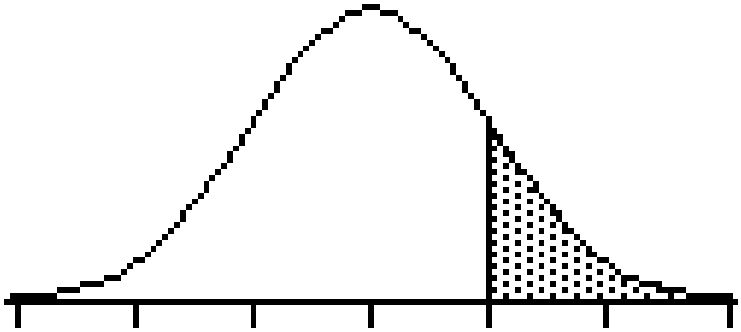
What percent of the population has an IQ above 120?

Area under normal curve

=

Probability of achieving various scores

What we need to do is find the area under the normal curve in the tail beyond an IQ or 120.



Theory: Calculus (antiderivative) gives area under normal curve between two points.

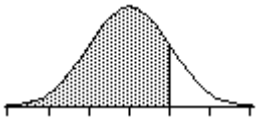
Good news: Somebody's already done it for you.

- The results are given as tables in your book.

Useful things to know:

- The **whole** normal curve has an area of **1** (or 100% of the data)
- Each half of the normal curve has an area of **.5** (or 50% of the data)

Your book has two tables –

- the "TAIL" table 
- the "BIG" table 

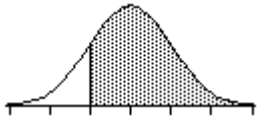
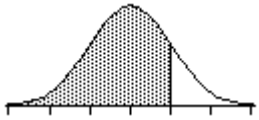
TYPES OF PROBLEMS

"Tail" Problems

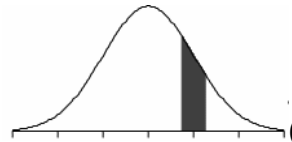
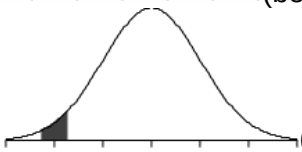
- z > POSITIVE 
- z < NEGATIVE 
- Just look up in "tail" table.

- Example: z > 1.72
- Example: z < -2.33

"Over Half" Problems

- z > NEGATIVE 
- z < POSITIVE 
- Just look up in "big" table.
  - Example: z > -1.23
  - Example: z < 2.07

"Same Side" Problems

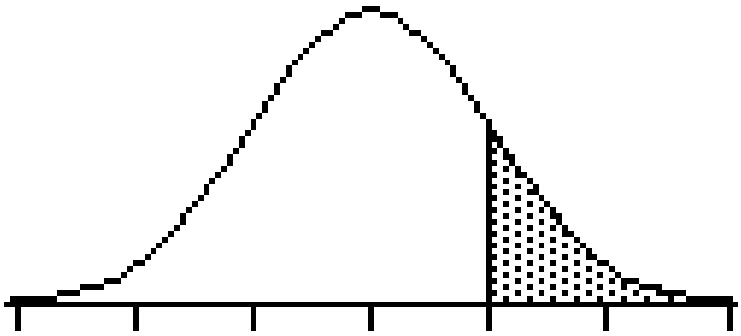
- positive < z < positive  (between two positive numbers)
- negative < z < negative  (between two negative numbers)
- Look up both numbers in either table.
- Subtract (BIG – SMALL) to get answer.
  - Ex.: 0.45 < z < 1.93
  - Ex.: -2.44 < z < -1.60
  - Ex.: 0 < z < 1.54

"Both Sides" Problems

- negative < z < positive  (between a negative and a positive)
- Look up both numbers in "tail" table
- Subtract both tails from 1 … 1 – FIRST – SECOND
  - Ex.: -1.28 < z < 0.55

Back to the original problem…

What we need to do is find the area under the normal curve in the tail beyond an IQ or 120.



FIRST, find the z-score associated with a an IQ of 120. (To do this, you need to know that for IQ $\bar{x}$ =120 and s=15.)

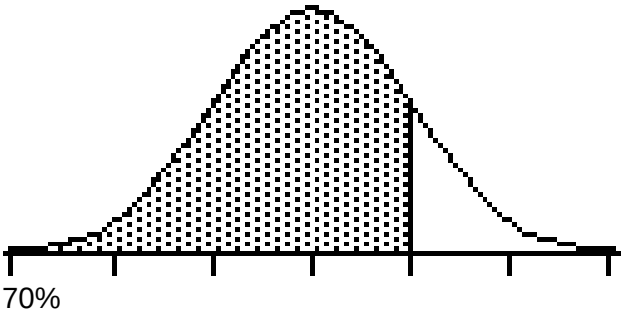
… So $z = \frac{(120 - 100)}{15} = 1.33$

NOW, find the percent of scores that are greater than 1.33 (look up in "tail" table).

- .0918 … so about 9%.

Sometimes problems are presented backwards.

Find "z" so that 70% of all scores are less than "z".



Look through the columns in the "big" table for the number closest to .7000 .

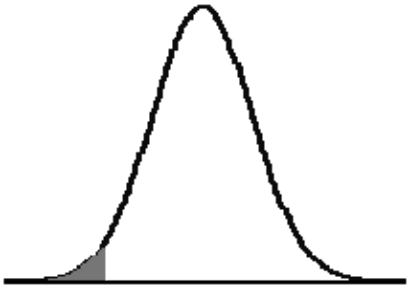
- The two closest are .6985 and .7019 .
- .6985 is the closest.
- The associated z-score is **0.52**, which is the answer.

Back to the IQ Example

- For IQ, the average is 100, and the standard deviation is 15.

```
IQ of 70
     z = (70-100)/15 = -2.00
IQ of 80
     z = (80-100)/15 = -1.33
IQ of 71
     z = (71-100)/15 = -1.93
```

We want the probability z is **LESS** than each of these numbers.



They are all TAIL problems.

P(z < -2.00) = **.0228**
P(z < -1.33) = **.0918**
P(z < -1.93) = **.0268**

Back to the ACT/SAT Example

For ACT …
Mean = 19.2
S.D. = 5.7

For new SAT …
Mean = 1511
S.D. = 290

JOLENE (25 ACT)
z = (25 – 19.2)/5.7 = 1.02

ROGER (1600 SAT)
z = (1600 – 1511)/290 = 0.31

Jolene's z-score is higher, so she did better.