Chapter 11

Counting & probability

Counting

- In advanced math "counting" means finding how many ways something can happen.
- We care about counting when it's something we can't count by hand.

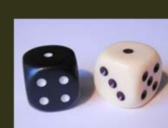
Fundamental Principle of Counting

- If one event can happen in "x" ways and another event can happen in "y" ways, then the 2 events can happen together in x•y ways.

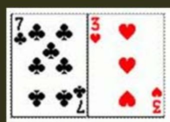
- When more than one thing happens at once, **multiply** to find the total possible outcomes.

EXAMPLE
If you roll two dice, how many ways could they land?

6 x 6 = **36**

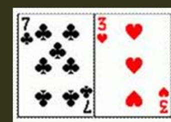


EXAMPLE
You draw a card from a deck of cards, put it aside, and draw another card.

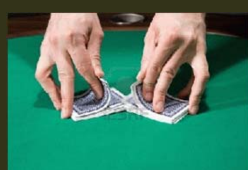


How many ways can you do this?

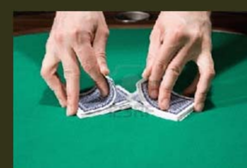
52 x 51 = **2652**



You draw a card from a deck of cards, put it back and re-shuffle, and then draw another card.

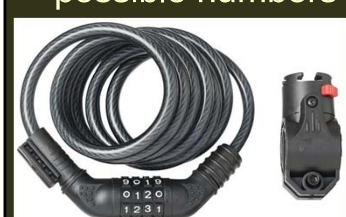


How many ways can you do this?



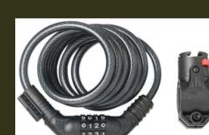
52 x 52 = **2704**

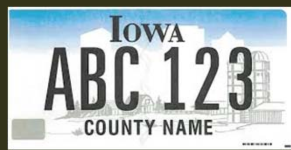
A bicycle lock has a 4-digit combination. How many possible numbers are there?



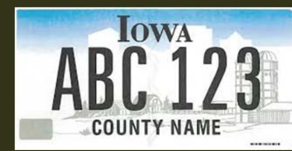
A bicycle lock has a 4-digit combination. How many possible numbers are there?

10 x 10 x 10 x 10
10^4 = **10,000**





EXAMPLE
As of 2015, most Iowa license plates have the format "ABC 123". How many plates are possible with this format?



26 x 26 x 26 x 10 x 10 x 10

17,576,000

If you take a true/false test with 12 questions, how many ways could you answer the questions?

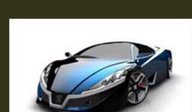
If you take a true/false test with 12 questions, how many ways could you answer the questions?

2 x 2 x 2 x 2
2 x 2 x 2 x 2 x
2 x 2 x 2 x 2
or 2^{12} = **4096**

A car is available with

- 3 body styles
- 2 trim lines
- 5 colors
- 2 types of transmissions
- 4 options packages

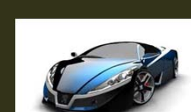
How many versions of the car are there?



A car is available with

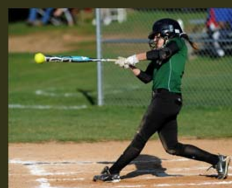
- 3 body styles
- 2 trim lines
- 5 colors
- 2 types of transmissions
- 4 options packages

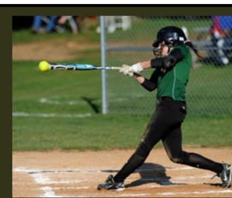
3 x 2 x 5 x 2 x 4
240



There are 9 starting players on a softball team.

How many batting orders are possible?





9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1
= **362,880**

This is also called **9-factorial**, which is written **9!**

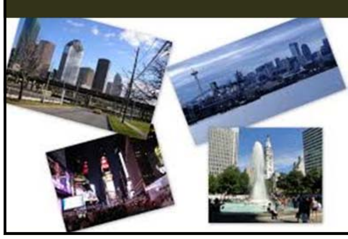
n! = n(n – 1)(n – 2) … (1)

Multiply a number by every natural number less than it.

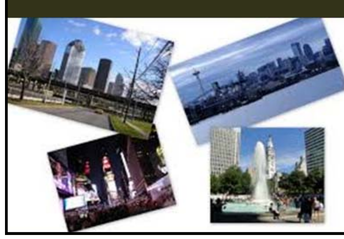
So

7! = 7 • 6 • 5 • 4 • 3 • 2 • 1
= **5040**

If a salesperson needs to visit 4 cities, in how many different orders could this be done?



If a salesperson needs to visit 4 cities, in how many different orders could this be done?



4!
4x3x2x1
24

Simplifying Factorial Expressions

- In most cases you can simplify factorial expressions by cancelling.

Ten students have to give presentations in a class. One of them insists on going first. If this request is granted, how many orders could the presentations be done in?

Basically we have the 1st person and the other 9.

There is only 1 way the first person can go.

The other 9 are 9!

It's the same as the softball problem … **5040**.

Example, simplify $\frac{84!}{80!}$

The numbers are too big to just type into most calculators.

However, this is just

$$\frac{84 \bullet 83 \bullet 82 \bullet 81 \bullet 80 \bullet 79 \bullet 78 \bullet \ldots}{80 \bullet 79 \bullet 78 \bullet \ldots}$$

Everything from 80 down is the same top & bottom and cancels.

So …

$$\frac{84!}{80!} = \frac{84 \bullet 83 \bullet 82 \bullet 81 \bullet 80 \bullet 79 \bullet 78 \bullet \ldots}{80 \bullet 79 \bullet 78 \bullet \ldots} = 84 \bullet 83 \bullet 82 = \underline{\underline{571{,}704}}$$

Permutations
How many **ORDERS** you can choose a small group from a big group.

EXAMPLE:
Eight people are running a race. In how many ways could medals for 1st, 2nd, and 3rd place be awarded?

This is $_8P_3$ or 8 nPr 3.

You can work permutations on many calculators or with a formula.

The formula is $_nP_r = \frac{n!}{(n-r)!}$

Here you'd take 8! ÷ (8 – 3)!

This is 8! ÷ 5!

It simplifies to 8 • 7 • 6 = **336**

DISTINGUISHABLE PERMUTATIONS

e.g. How many ways could you shuffle the letters in the word HELLO and tell the new words apart?

HELLO and LELHO are distinguishable permutations, but ELOLH and ELOLH (switching around the 2 L's) aren't.

To find the number of distinguishable permutations in HELLO …

$$\frac{5!}{2!}$$

5! ÷ 2! = **60**

To find the number of distinguishable permutations in ALABAMA …

$$\frac{7!}{4!}$$

7! ÷ 4! = **210**

To find the number of distinguishable permutations in EMMETSBURG …

$$\frac{10!}{2! \bullet 2!}$$

On most calculators, all you really have to do is divide by each factor on the bottom

$$\frac{10!}{2! \bullet 2!}$$

10! ÷ 2! ÷ 2!

= **907,200**