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- Earlier we learned about permutations, which meant choosing a small group out of a large group in different orders.
- Combinations also involve choosing a small group out of a larger group.
- The difference is that with combinations you don't care about the order.

COMBINATIONS—it doesn't matter what order you choose things in

PERMUTATIONS—the order matters

Notation:

- nPr stands for permutations
- nCr stands for combinations

On a graphing calculator, for both permutations & combinations

- Put the big number on the screen.
- Select the **MATH** menu.
- Select **PRB** for "PROBABILITY".
- Select either **nCr** or **nPr.**
- Hit **ENTER**.
- Put the small number on the screen, and hit **ENTER** again.

Many scientific calculators also do permutations and combinations.

- Check the instructions.

You can also work combinations through a formula.

$$n\ C\ r = \frac{n!}{r!(n-r)!}$$

Example:
How many ways can you 6 lottery numbers be chosen from 60 possible numbers?

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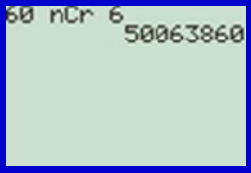
- It doesn't matter what order the numbers are in, just whether they match the ones you picked.
- So, this is problem involves combinations.

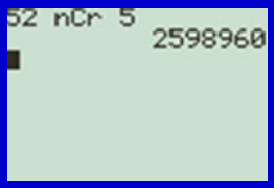


The answer is **50,063,860**

Example:
How many 5-card poker hands are possible when you use a 52-card deck?

This is also a combinations problem.

The problem is 52 nCr 5.



2,598,960

Example

How many ways can a hotel hire 3 maids from 13 applicants?

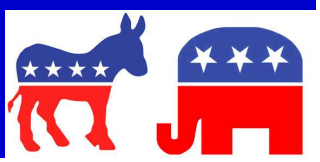
This is 13 nCr 3 .

$$13\ nCr\ 3 = \frac{13!}{3!\,(10)!}$$

$$\frac{13 * 12 * 11 * 10 * 9 * 8 \ldots}{3 * 2 * 1 * (10 * 9 * 8 \ldots)}$$

$= \frac{13*12*11}{3*2*1} = \frac{1716}{6}$ = 286 ways

Example:
The Iowa House consists of 57 Republicans and 43 Democrats. How many ways could a committee of 3 Republicans and 2 Democrats be chosen?



3 Republicans … 57 nCr 3
29,260
2 Democrats … 43 nCr 2
903

3 Republicans … 57 nCr 3
29,260
2 Democrats … 43 nCr 2
903

3 Republicans **AND** 2 Democrats … **MULTIPLY**
29,260 X 903 = **26,421,780**

Example:
At a banquet, you may choose beef, pork, chicken, or fish for your entrée. You then choose 2 sides from among beans, corn, asparagus, potato, rice, and carrots. How many ways could you order the meal?

Entrée
4 choices
Side dishes
6 nCr 2 = 15

Entrée AND side dishes
4 x 15 = 60

Probability
- fraction that tells how likely something is to happen

- Notation:
  P(x) means the probability of event "x".
- Probability is always a fraction between 0 & 1

- If P(x) = 0, then the event is **impossible** (can't ever happen).
- If P(x) = 1, then the event is **certain** (must always happen).

- Probability can also be expressed as a percent (which is officially called chance)
- Chance must always be between 0% and 100%.

Basic rule for finding probability:

$$P(x) = \frac{\text{Desirable Outcomes}}{\text{Total Outcomes}}$$

$$P(x) = \frac{\text{Ways to get what you want}}{\text{Ways to get anything}}$$

$$P(x) = {}^{x}/_{n}$$

If you roll a die, what is the probability of rolling
- the number 4?
- an odd number?
- a number greater than 2?

If you roll a die, what is the probability of rolling
- the number 4?

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1 out of 6 or $^1/_6$

If you roll a die, what is the probability of rolling
- an odd number?

If you roll a die, what is the probability of rolling
- an odd number?

3 out of 6 = $^3/_6$ or $^1/_2$

If you roll a die, what is the probability of rolling
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  So 3, 4, 5 or 6

If you roll a die, what is the probability of rolling
- a number greater than 2?
  So 3, 4, 5 or 6

4 out of 6 = $^4/_6$ or $^2/_3$

EXAMPLE:

One version of the Magic 8-Ball has 20 different responses …

9 “yes”
8 “no”
3 unresponsive

If you ask the Magic 8-Ball a question, what is the probability the answer is equivalent to “yes”?

9 “yes”
8 “no”
3 unresponsive

So $^{9}/_{20}$ or .45

EXAMPLE:

In a standard deck there are 52 cards.

If you pick a card at random, what is the probability …

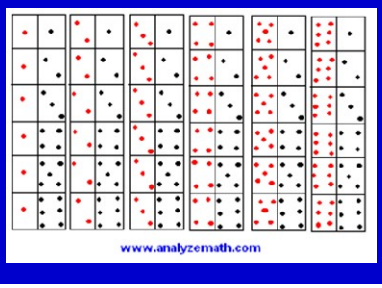
* It is a king?

* It is a spade?

* It is the queen of hearts?

EXAMPLE:
If you roll 2 dice,
what is the probability you get a total of 8?

On dice there are 6 x 6 or 36 possible outcomes …



To get "8", you could have …

4 – 4
5 – 3
3 – 5
2 – 6
6 – 2

So it's 5 out of 36 … $^{5}/_{36}$

EXAMPLE:

When you flip a coin, what is the probability you get "heads"?

Theoretical Probability

- All the examples we've done have given theoretical probability.
- This is a mathematical prediction of what the probability should be
- Never changes

Empirical Probability

- a.k.a. "Experimental Probability"
- Actual results, based on past experience
- Can change with circumstances.
- Rarely exactly equals the empirical probability.

Law of Large Numbers

- If an event happens over and over again, the empirical probability will approach the theoretical probability

This is why while some people win big in casinos, in the end the house will always come out ahead.

Compound Probability

AND, OR, *and* ***NOT***

NOT
(also called "Complement")

- P(NOT x) = 1 – P(x)
- So if P(x) = ¼, then P(NOT x) = ¾.

We do this all the time with percents (where you subtract from 100%).

If there's a 30% chance of rain tomorrow, there's a 70% chance it **WON'T** rain.

EXAMPLE:

One version of the Magic 8-Ball has 20 different responses …

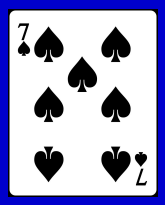
9 "yes"
8 "no"
3 unresponsive

If you ask the Magic 8-Ball a question, what is the probability the answer is NOT "no"?

9 "yes"
8 "no"
3 unresponsive

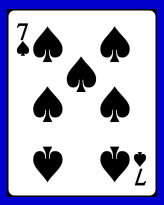
$(1 - {}^{8}/_{20}) = {}^{12}/_{20} = .6$

EXAMPLE:
If you draw a card out of a deck of cards, what is the probability it is NOT a diamond?



EXAMPLE:
If you draw a card out of a deck of cards, what is the probability it is NOT a diamond?

$1 - {}^{13}/_{52} = {}^{39}/_{52}$ or $3/4$



If there is an 85% chance of windy weather tomorrow, what is the probability it won't be windy?

If there is an 85% chance of windy weather tomorrow, what is the probability it won't be windy? 100 – 85 = **15%**

AND

- When more than one thing happens at once, **multiply** to find the total possible outcomes.

If you flip a coin and roll one die, what is the probability you get "heads" and a "3"?

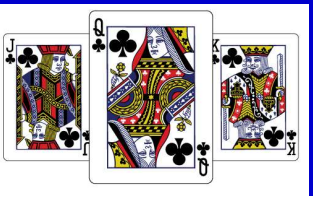
$^1/_2$ X $^1/_6$ = $^1/_{12}$

OR

In most cases

P(A or B) = P(A) + P(B)

If you draw a card out of a deck of cards, what is the probability you get a face card or an ace?



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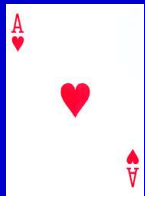
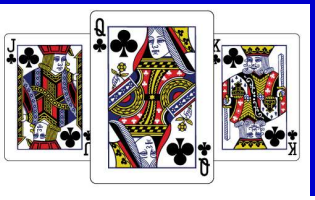
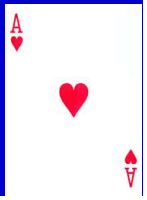


$^{12}/_{52}$ + $^4/_{52}$ = $^{16}/_{52}$ or $^4/_{13}$





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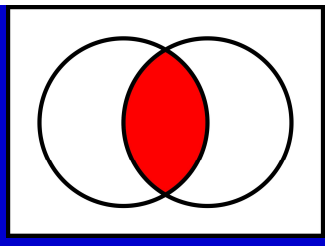


If the events can overlap,
P(A or B) =
P(A) + P(B) – P(A and B)

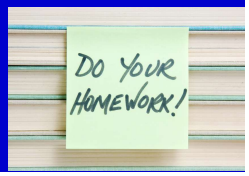
- Probability of 1st
  PLUS
- Probability of 2nd
  MINUS
- Probability BOTH will happen together

There is a 70% chance Joe will watch TV tonight.

There is a 40% chance Joe will do his homework tonight.

There is a 25% chance Joe will both watch TV and do his homework.

What is the probability Joe will either watch TV or do his homework tonight?



70% TV
40% homework
25% TV and homework

70 + 40 – 25 = **85%**

Remember …
In probability
and means **times**
or means **plus**

Odds

- The likelihood of something, expressed as a ratio

Most often we care about the odds **AGAINST** something happening.

- To find the odds against, take the number of ways something WON'T happen to the number of ways it will happen.
- Bad : Good
- P(not X) : P(X)

- It's also possible to find the odds in favor of something.
- This is the exact opposite.

  Good : Bad

  P(X) : P(not X)

Example:
The probability of winning a raffle is 1 out of 500.

Find the odds against winning and the odds in favor of winning.

Odds Against
499 : 1

Odds in Favor
1 : 499

Example:
Find the odds against rolling an 11 when you roll 2 dice.

To get an 11 you must get a 5 – 6 or a 6 – 5 .

So there are 2 (out of 36) ways you can win.

This means there are 34 (out of 36) ways you can lose.

So the odds against are
34 : 2 or 17 : 1

Expected Value

- Using probability to predict an average

To find expected value …

$\Sigma x \cdot P(x)$

- Multiply each value times the probability of getting that value.
- Add up the products.

Example:
Suppose the odds of winning a particular lottery game are as follows:

1 in 25,000	wins	$5,000
10 in 25,000	wins	$1,000
100 in 25,000	wins	$100
1000 in 25,000	wins	$10
10,000 in 25,000	wins	$1

What are the average winnings of people who play this lottery game?

1 in 25,000	wins	$5,000
10 in 25,000	wins	$1,000
100 in 25,000	wins	$100
1000 in 25,000	wins	$10
10,000 in 25,000	wins	$1

On your calculator, enter:

```
 1 / 25000 * 5000 + 10 /
25000 * 1000 + 100 / 25000
* 100 + 1000 / 25000 * 10 +
10000 / 25000 * 1
```

= **1.8**

The expected value is $1.80