

## Number theory

- studying properties of the natural numbers

## Natural Numbers

- 1, 2, 3, 4, 5, 6 ...
- numbers you count with
- positive (not zero) whole numbers

## Whole numbers

- 0, 1, 2, 3, 4, 5, ...
- the natural numbers, and also zero.
- No negatives; no fractions

## Integers

- ... -3, -2, -1, 0, 1, 2, 3, ...
- Whole numbers and their opposites

Each of these sets includes the ones from before

Every natural number is also a whole number and an integer.

## Absolute value

- Distance a number is from zero
- Can't be negative
- Symbol ...  $|x|$  means "the absolute value of x"

Examples:

$$|-3| = 3$$

$$|2| = 2$$

$$|0| = 0$$

## Relationship Symbols

<                      →                      Less than

>                      →                      Greater than

=                      →                      Equal

If a number is less, it is to the left on a number line or to the bottom on a thermometer.

If a number is greater, it is to the right on the number line or to the top on a thermometer.

Place <, >, or =

$$2 \quad \underline{\quad} \quad -5$$

$$-7 \quad \underline{\quad} \quad -3$$

$$-5 \quad \underline{\quad} \quad 0$$

$$0 \quad \underline{\quad} \quad 4$$

$$7 \quad \underline{\quad} \quad 5$$

$$-2 \quad \underline{\quad} \quad 3$$

### Using calculators with negative numbers

On graphing calculators, the key marked  $\boxed{-}$  means “negative”.

On older cheap calculators, the key marked +/- means “negative”.

Things it's useful to know about negative numbers:

- Neg X Neg = Pos
- Neg  $\div$  Neg = Pos
- Pos X Neg = Neg
- Pos  $\div$  Neg = Neg
- Neg X Pos = Neg
- Neg  $\div$  Pos = Neg
- Pos X Pos = Pos
- Pos  $\div$  Pos = Pos

... So if you multiply or divide numbers with the **same sign**, the answer is **positive**.

...If you multiply or divide numbers with **opposite signs**, the answer is **negative**.

### Special Cases

- $0 \times$  anything = 0
- $0 \div$  anything (besides 0) = 0
- anything  $\div 0$  is **undefined** (can't do it)

### TAKING POWERS

- On graphing calculators, the power key looks like  $\boxed{\wedge}$ .
- On older cheap calculators, the power key looks like  $\boxed{y^x}$ .

### Important ...

- When you take powers of negative numbers, always put the number in parentheses.
- A negative number to an **even** power will have a **positive** answer.
- A negative number to an **odd** power will have a **negative** answer.

### Order of Operations

What order you should do things when evaluating an expression

- Parentheses (and other grouping symbols)
  - Brackets
  - Top or bottom of fraction
  - Work inside out, then left to right if more than one.
- Exponents
  - Work left to right if more than one
- Multiplication & division
  - Work left to right (regardless of which comes first)
- Addition & subtraction
  - Work left to right (regardless of which comes first)

## Exponents ... Powers

- When you take a number to a power, you multiply it by itself repeatedly.
- $3^5 = 3 \bullet 3 \bullet 3 \bullet 3 \bullet 3 = 243$
- $2^7 = 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 = 128$
- $(-4)^3 = (-4)(-4)(-4) = -64$
- $(-3)^2 = (-3)(-3) = 9$
- $0^8 = 0 \bullet 0 \bullet 0 \bullet 0 \bullet 0 \bullet 0 \bullet 0 \bullet 0 = 0$

Write  $7 \bullet 7 \bullet 7 \bullet 7 \bullet 7$  with exponents

...  $7^5$

Rules for working with exponents:

### Product Rule

- $x^n \bullet x^m = x^{n+m}$
- When you multiply things with exponents, **add** the exponents.
- $3^2 \bullet 3^4 = 3^6$
- $(5^9)(5^3) = 5^{12}$
- $n^8 \bullet n^8 = n^{16}$

### Quotient Rule

- $\frac{x^n}{x^m} = x^{n-m}$
- When you divide or make a fraction out of things with exponents, **subtract** the exponents.
- $\frac{2^4}{2^2} = 2^2$
- $5^9 \div 5^3 = 5^6$
- $\frac{7^5}{7^4} = 7^1$  ... or just 7

### Power Rule

- $(x^n)^p = x^{np}$
- When you raise a power to a power, **multiply** the exponents.
- $(5^3)^2 = 5^6$
- $(8^9)^5 = 8^{45}$
- $(2^2)^4 = 2^8$

### Zero Exponent Rule

- $x^0 = 1$
- If you raise anything (except 0) to the zero power, the answer is always 1.
- $3^0 = 1$
- $5^0 = 1$
- $10^0 = 1$

## Negative Exponent Rule

- $x^{-m} = \frac{1}{x^m}$
- When you take something to a **negative** power, it makes a fraction (reciprocal).
- $3^{-2} = \frac{1}{9}$
- $2^{-3} = \frac{1}{8}$

## Scientific Notation

- a shorthand way to write very large or very small numbers
- In scientific notation, numbers always have the form \_\_\_\_ X 10<sup>...</sup>.

To change a number into scientific notation ...

- Move the decimal so there is just one place before it.
- Count the places after the decimal

Example:

Change 53,700,000,000 to scientific notation  
 $5.37 \times 10^{10}$

Example:

Change 435,300,000 to scientific notation  
 $4.353 \times 10^8$

- If the number is already a decimal, you still move the decimal so there is just one place before it.
- Count how many places you moved the decimal; the exponent is **negative** that number. (This is always one more than the number of 0's after the original decimal.)

Example:

Change .000412 to scientific notation.  
 $4.12 \times 10^{-4}$

Example:

Change .00000000000024 to scientific notation  
 $2.4 \times 10^{-13}$

To change back to decimal notation ...

- Copy the significant digits
- If the exponent is positive, there are that many places after the first digit; add zeros to make the number of places.
- If the exponent is negative, put in one fewer zeros than the exponent at the beginning.

Example:

Change  $3.7 \times 10^5$  to decimal notation.  
370,000

Example:

Change  $5.417 \times 10^{12}$  to decimal notation.  
5,417,000,000,000

Example:

Change  $3.4 \times 10^{-5}$  to decimal notation.

.000034

Example:

Change  $2.456 \times 10^{-7}$  to decimal notation

.0000002456