Sequences

- A list of numbers that makes a pattern

Arithmetic sequence

- You add the same number each time to get the next number in the sequence
- 5, 8, 11, 14, 17, 20, …
- 3, 7, 11, 15, 19, 23, …
- 52, 47, 42, 37, 32, …

- The number you add each time is called the **common difference** or "d".

NOTATION … a_1 means the first term of a sequence, a_2 means the second term, etc.

So in 5, 8, 11, 14, 17, 20,

$$a_4 = 17$$

- You can find **any** term of any arithmetic sequence by using the formula

$$a_n = a_1 + d(n - 1)$$

Example: Find the 32nd term of the sequence 5, 9, 13, …

$a_{32} = 5 + 4(32 - 1)$
$= 5 + 4 \bullet 31$
$= 129$

Example: Find the 10th term of the sequence 100, 90, …

$a_{10} = 100 + (-10)(10 - 1)$
$= 100 + (-10)9$
$= 10$

Geometric sequence

- You multiply by the same number each time to get the next number in the sequence
- 5, 10, 20, 40, 80, …
- 3, 9, 27, 81, …
- 625, 125, 25, 5, 1, $^1/_5$, …
- The number you multiply by each time is called the **common ratio** or "r".
- You can find **any** term of any arithmetic sequence by using the formula

$$a_n = a_1 r^{n-1}$$

Example: Find the 10th term of the sequence 3, 6, 12, …

$a_{10} = 3 \bullet 2^{10-1}$
$= 3 \bullet 2^9$
$= 1536$

Example: Find the 6th term of the sequence 1000, 500, 250, …

$a_6 = 1000(½)^{6-1}$
$= 1000(½)^5$
$= 31.25$

So far we have learned …

Natural numbers ($\mathbb{N}$)
- { 1, 2, 3, 4, 5, … }
- numbers you count with
- always positive
- never fractions

Whole numbers
- { 0, 1, 2, 3, 4, … }
- natural numbers, and also zero

Integers
- { … -3, -2, -1, 0, 1, 2, … }
- whole numbers and their opposites

Every natural number is also a whole number and an integer.

Other sets of numbers …

Rational numbers
- "ratio" means fraction
- Rational numbers include anything that can be written as a fraction of integers.
- ¾, -½, 2¼, -.5, .4, 7.3
- Integers like 6, -3, and 0 are also rational numbers.

- Rational numbers can always be expressed as a decimal which either terminates (ends) or repeats.

Express $^{5}/_{11}$ as a decimal.
- Just take 5 ÷ 11
- = .384615384615…

Express .72 as a fraction.

$^{72}/_{100} = {}^{36}/_{50} = {}^{18}/_{25}$

Most calculators have features that help you work with fractions.

- Cheap calculators normally have a fraction key that looks like $a^b/_c$.
- You can enter ¾ by typing 3 $a^b/_c$ 4.
- You can use this key to reduce fractions (hit = after entering a fraction) and to do math with fractions.

- Graphing calculators have a feature that will change decimals to fractions.
- On the TI-83, you type a decimal and hit MATH then ENTER twice.

Example: change .725 to a fraction …

.725 MATH ENTER ENTER

```
.725►Frac
                                29/40
```

Example: change $.\overline{38}$ to a fraction. …

.38383838383838
MATH ENTER ENTER

(Make sure you go all the way across the screen with the decimal.)

```
.3838383838►
Frac
                              38/99
```

You can also use the "►Frac" feature to do math with fractions.

- Use ÷ for the fraction bar.
- Type in the problem. Then hit MATH ENTER ENTER at the end.

Example: $^{3}/_{5} + {}^{1}/_{8}$ …

```
3/5+1/8►Frac
                              29/40
```

Irrational Numbers

- **NOT** rational
- Numbers that CAN'T be written as a fraction of integers
- "Weird" numbers
- Non-terminating, non-repeating decimals

Examples of irrational numbers:

- Special numbers like π
- Roots that are not whole numbers like $\sqrt{7}$
- Decimals that don't repeat the **exact** same thing like .34334433344433334444…

The most common irrational numbers we use are square roots.

Almost every calculator can work with square roots, though sometimes you need to press the INV or 2nd key first.

- On a graphing calculator, to enter $\sqrt{13}$, type $\sqrt{\ }$ (which is 2nd and x^2) and then 13. Press ENTER to get the answer.

```
√(13
                          3.605551275
```

- Some calculators will show parentheses after you hit $\sqrt{\ }$. Others will show a "PrettyPrint" screen that shows the number under the root.
- On most scientific calculators, to enter $\sqrt{13}$, type 13 and then $\sqrt{\ }$ (which may require the INV or 2nd key). The answer should appear as soon as you hit $\sqrt{\ }$.

Real Numbers

- **ALL** numbers
- Both rational and irrational numbers together

Tell which numbers in this set are …

- Natural
- Whole
- Integers
- Rational numbers
- Irrational numbers
- Real numbers

{ 5, -2 $^{3}/_{8}$, $\sqrt{5}$, $-\sqrt{16}$, .283, -5.5 }

Properties of real numbers

- Things that will always be true for all real numbers.

Commutative Property

- 5 + 4 = 4 + 5
- 7 x 3 = 3 x 7
- You can multiply or add in any order, and it doesn't change the answer.

Associative Property

- (3 + 4) + 1 = 3 + (4 + 1)
- 4(6 x 3) = (4 x 6) x 3
- You can group together what you want when you add or multiply.

Distributive Property

- 3(2x + 7) = 6x + 21
- 5(3x – 2) = 15x – 10
- -4(2x – 1) = -8x + 4
- If you take a number times something in parentheses, multiply what's in front times each thing in ( ), one at a time.